Persistent currents in quasi-one-dimensional quantum rings: analytical treatment of the interplay of the interelectron Coulomb interaction and disorder at finite temperatures

1997 J. Phys.: Condens. Matter 9 1521

(http://iopscience.iop.org/0953-8984/9/7/015)

J. Phys.: Condens. Matter **9** (1997) 1521–1536. Printed in the UK PII: S0953-8984(97)70484-3

Persistent currents in quasi-one-dimensional quantum rings: analytical treatment of the interplay of the interelectron Coulomb interaction and disorder at finite temperatures

Alexander S Rozhavsky†

Centre de Recherches sur les Tres Basses Temperatures, associé a l'Université Joseph Fourier, CNRS, BP166, 38042 Grenoble Cédex 9, France

Received 2 January 1996, in final form 16 September 1996

Abstract. We study persistent currents in a gas of interacting electrons confined to a quasi-one-dimensional disordered ring at arbitrary temperatures in two experimentally available regimes: at a fixed chemical potential and at a fixed number of particles. In a realistic experiment, the interelectron Coulomb interaction in a gated ring is unscreened and the potential of impurities is a smooth function of coordinates. The latter allows us to study the persistent current non-averaged over impurity realizations in a small ring. In a quasi-one-dimensional electron gas, only forward- and back-scattering processes are relevant. The forward-scattering amplitude of the Coulomb field enhances the persistent current at a fixed chemical potential but does not affect it at a fixed number of particles. The Coulomb backscattering suppresses the intrinsic disorder and enhances the persistent currents at both a fixed chemical potential and a fixed number of electrons. Increasing the number of transverse channels enhances the persistent current at strong Coulomb repulsion: $e^2/\hbar V_F \gg 1$.

Strong Coulomb interaction cancels out oscillations over the electrochemical potential. The parity effect at a fixed number of particles is defined by the average occupation of a single channel.

The non-monotonic temperature dependence of a persistent current is predicted.

1. Introduction

Persistent currents in thin conducting rings pierced by magnetic flux are the manifestation of the Aharonov–Bohm effect (ABE) in many-particle systems [1, 2]. The wavefunction of the conduction electron, which encircles the ring, acquires the phase shift

$$\Delta\Theta_{A-B} = 2\pi\frac{\Phi}{\Phi_0}$$

where Φ is the magnetic flux and $\Phi_0 = hc/e$ is the magnetic flux quantum. As a result, all the thermodynamic quantities oscillate with the flux, the period of oscillations being Φ_0. The persistent current $J(\Phi)$ is the derivative of the free energy over the magnetic flux:

$$J(\Phi) = -c\frac{\partial F}{\partial \Phi} \tag{1}$$

† Permanent address: B I Verkin Institute for Low Temperature Physics and Engineering, 47 Lenin Avenue, 310164 Kharkov, Ukraine.

0953-8984/97/071521+16$19.50

i.e. it is the oscillatory component of the magnetic moment. In non-superconducting rings, the ABE is mesoscopic; the persistent current magnitude tends to zero when the ring perimeter L increases to infinity. The thermodynamics of persistent currents in a one-dimensional impurity free ring containing a gas of non- interacting electrons was first studied in [3]. Later, in a series of papers [4], the effect of impurities on the persistent current in a free-electron gas was studied within the diffusion approximation. This approach was believed to describe a multichannel ring the width of which was large compared with the conduction electron wavelength. The experiments [5, 6] demonstrated, however, that the free-electron model failed to describe adequately the persistent current in a disordered ring, and it has been understood [4] that the Coulomb electron–electron interaction has to be incorporated non-perturbatively into the theory to match it to reality.

A theory of persistent currents for correlated electrons in dirty rings is still being considered; the majority of results are obtained for various models of one-dimensional (single-channel) rings at zero temperature [7–11, 14, 15, 24].

In the meantime, there does not exist a commonly accepted point of view on the interplay of Coulomb interaction and disorder; Abrahams and Berkovitz [7] claimed that the persistent current is either enhanced or suppressed depending on the ratio of the Coulomb parameter to the strength of disorder; Bouzerar *et al* [8] stated that the interelectron interaction always suppresses the persistent current; Chakraborty and Pietilainen [9] and Chakraborty [10] declared that the Coulomb parameter does not affect persistent currents at all; it has been shown analytically in [11] and numerically in [15, 24] that interaction enhances the persistent current.

All the above-cited papers treated the problem at zero temperature. The effect of the Coulomb interaction and a single local impurity on a persistent current at finite temperatures was studied in [28] for a one-dimensional Wigner crystal.

In this paper we present a theory of persistent current in a two-dimensional gas of interacting electrons confined to a disordered ring of finite width at finite temperatures. The model chosen is believed to describe adequately the low-channel ring formed in an $Al_xGa_{1-x}As/GaAs$ heterostructure [16]. The disorder potential is supposed to be a smooth function of coordinates because of a large concentration of impurities. We have obtained analytical results in the limit of a small number of channels at a strong Coulomb repulsion ($e^2/\hbar V_F \gg 1$) at low ($T \ll T_0/\pi$) and high ($T \gg T_0/\pi$) temperatures, where $T_0 = \hbar V_F/L$ is the spacing between the quantized electron levels in a one-dimensional ring [3, 4].

We have studied two realistic situations: firstly a fixed chemical potential and secondly a fixed number of particles in a ring. In the first case, both energy and particle exchange with the reservoir is allowed. The second limit corresponds to an isolated ring. In practice, the choice between these possibilities is regulated by the ratio of the mean level spacing ε to the normalized capacitance energy e^2/L; when $L\varepsilon/e^2 \geqslant 1$, particle exchange is allowed and, when $L\varepsilon/e^2 \ll 1$, it is suppressed (Coulomb blockade). Formally, these two limits correspond to a description within the grand canonical and the canonical ensembles, respectively.

In a quasi-one-dimensional conductor the Coulomb interaction is presented as a sum of forward- and back-scattering potentials. The strong forward scattering enhances the persistent current at a fixed chemical potential in a twofold way; it makes the current proportional to the number of transverse channels and increases the crossover temperature $T^*(> T_0/\pi)$. At a fixed number of particles, the forward scattering does not affect the persistent current.

The Coulomb back scattering increases the persistent current independent of the choice of ensemble; it suppresses the intrinsic localization of electrons and creates the new

temperature-dependent localization length which is much larger than the bare value. At $T > T_C$, T_C is a function of Coulomb and disorder parameters; the persistent current reaches its value in a pure metal.

Increasing the number of transverse channels enhances the persistent current at a strong Coulomb interaction.

Strong Coulomb fluctuations suppress oscillations over the electrochemical potential. At a fixed number of particles, the parity effect is defined by the occupation of a single channel.

2. Mathematical formalism: the model

We study the two-dimensional electron gas in a ring-shaped conductor of a radius R and width $d (r \ll R)$, created from a gated semiconductor heterostructure and placed in a vector potential. In such a system the interelectron long-ranged Coulomb interaction is relevant.

When $k_F d \approx 1$, the electron gas is of a quasi-one-dimensional nature; $M \approx 1$, where M is the number of zeros of the radial component of the wavefunction (the number of channels).

First of all, derive the effective Hamiltonian for quasi-one- dimensional electrons starting from the Hamiltonian of electrons on a lattice:

$$H = t_0 \sum_{r,\sigma} (a^+_{r\sigma} a_{r+l\sigma} e^{i\alpha} + \mathrm{HC}) + \sum_{r,\sigma} V_r a^+_{r\sigma} a_{r\sigma} + e^2 \sum_{\substack{r,r' \\ \sigma,\sigma'}} \frac{(a^+_{r\sigma} a_{r\sigma})(a^+_{r'\sigma'} a_{r'\sigma'})}{|\boldsymbol{r} - \boldsymbol{r}'|} - eV_g \sum_{r,\sigma} a^+_{r\sigma} a_{r\sigma}$$
$$= H_0 + H_{imp} + H_{Coulomb}. \tag{2}$$

Here the first term describes intersite electron hopping, t_0 is the hopping amplitude and $a^+_{r\sigma}$ is the electron creation operator at the site $\boldsymbol{r}$ with the spin σ; the second term describes the scattering of electrons by impurities; the third term is the interelectron Coulomb interaction; $\alpha = (e\boldsymbol{A} \cdot \boldsymbol{l})/\hbar c$ and V_g is the gate voltage.

In the quasi-one-dimensional limit, all the electrons are classified in terms of right- and left-moving particles:

$$a_{mj} = \psi_R(mj) \exp(ik_F ma) + \psi_L(mj) \exp(-\mathrm{i}k_F ma) \tag{3}$$

where m is the site index along the perimeter of the ring and j that in the radial direction; a is the lattice spacing (in what follows, we put $\hbar = c = a = 1$).

The amplitudes $\psi_{R,L}$ are smooth functions of coordinates which allow us to linearize the electron spectrum in the vicinity of the points $k \cong \pm k_F$. In terms of the ψ fields the Hamiltonian H_0 takes the well known form (see, e.g., [17])

$$H_0 = \sum_{n,\sigma} \int \mathrm{d}x \left\{ \mu \bar{\psi} \gamma_0 \psi + \mathrm{i} V_F \bar{\psi} \gamma_1 (\partial_x - \mathrm{i}eA)\psi + \frac{\pi^2 n^2}{2m^* d^2} \bar{\psi} \gamma_0 \psi \right\}. \tag{4}$$

Here ψ denotes a spinor:

$$\psi = \begin{pmatrix} \psi_{R\sigma} \\ \psi_{L\sigma} \end{pmatrix} \tag{5}$$

$\psi_{R,L} = \psi_{R,L}(n, x)$, $\bar{\psi} = \psi^+ \gamma_0$, $(\gamma_0, \mathrm{i}\gamma_1)$ are Pauli matrices and $\mu = k_F V_F - eV_g$.

When deriving the low-channel representation of H_{imp} and $H_{Coulomb}$, one has to keep only Fourier harmonics with the wavevector $k \cong 0$ (forward scattering), and $k \cong \pm 2k_F$ (back scattering). The driving motive for such an oversimplification is that the impurity potential in the case of interest is created by scattering centres which lie outside the plane

of the two-dimensional gas, and their influence on the electronic states in the ring is solely through a long-range Coulomb potential. As a result, the impurity potential in the ring is smooth on the scale of the Fermi wavelength, and the Hamiltonian for electron–disorder interaction takes the form

$$H_{imp} = \sum_{n,\sigma} \int \mathrm{d}x \, (\mu_{imp} \bar{\psi} \gamma \psi - \Delta \bar{\psi} \psi) \tag{6}$$

where μ_{imp} is the forward- and Δ is the back-scattering amplitude. In what follows we study the realistic case of a high concentration of impurities when both μ_{imp} and Δ vary slowly on the ring length L, and thus the persistent current is not self-averaged over realizations of the impurity potential. We neglect the interchannel transitions.

When $M \approx 1$, the Hamiltonian of electron–electron interaction takes the form

$$H_{Coulomb} = e^2 \sum_{\substack{n,n' \\ \sigma,\sigma'}} \int \mathrm{d}x \int \mathrm{d}x' \, \frac{(\psi \bar{\gamma}_0 \psi)_{xn\sigma} (\psi \bar{\gamma}_0 \psi)_{x'n'\sigma'}}{|x - x'|}$$
$$+ e^2 \log(2k_F b) \sum_{\substack{n,\sigma \\ n',\sigma'}} \int \mathrm{d}x \, (\bar{\psi}\psi)_{n\sigma} (\bar{\psi}\psi)_{n'\sigma'} \tag{7}$$

where the first term on the right-hand side of (7) describes the forward scattering, and the second term the back scattering; $b \cong d$.

It should be noted that, when modelling the Coulomb interaction, we substitute the ring geometry by a line. Such a substitution can be justified only for a long ring, $L \gg d$.

2.1. Partition function: general formulae

It is convenient to present the grand partition function $Z(\mu)$ in the form of a path integral over the Grassman variables $\bar{\psi}_\sigma$ and ψ_σ [18]:

$$Z(\mu) = \int \mathrm{D}\bar{\psi} \, \mathrm{D}\psi \exp S = \int \mathrm{D}\bar{\psi}_\uparrow \, \mathrm{D}\psi_\uparrow \, \mathrm{D}\bar{\psi}_\downarrow \, \mathrm{D}\psi_\downarrow \exp S = Z_\uparrow Z_\downarrow \tag{8}$$

where S is the Euclidean action:

$$S = \sum_n \int_0^L \mathrm{d}x \int_0^\beta \mathrm{d}\tau \, \bar{\psi} \gamma_0 \partial_\tau \psi - H. \tag{9}$$

Here τ is the imaginary time and $\beta = 1/T$ is the inverse temperature.

We introduce integration over auxiliary fields [18] to decompose the four-fermion operators in the Coulomb Hamiltonian:

$$\exp S_{Coulomb} = \int \mathrm{D}\varphi \, \mathrm{D}\chi \exp \left(- \sum_n \int \mathrm{d}x \, \mathrm{d}\tau \left\{ [\varphi(\hat{O}_0)\varphi] - 2e\varphi \bar{\psi} \gamma_0 \psi + \frac{\chi^2}{g} + \chi \bar{\psi} \psi \right\} \right) \tag{10}$$

where

$$g = e^2 \log(2k_F b) \tag{11}$$

and the operator $\hat{O}_0$ is

$$\hat{O}_0 = x^2 \partial_x^2 + 2x \partial_x$$
$$\hat{O}_0 \frac{1}{|x|} = -2\delta(x). \tag{12}$$

Eventually, the partition function (8) takes the form of a Gaussian integral:

$$Z(\mu) = \int \mathrm{D}\bar{\psi}\,\mathrm{D}\psi\,\mathrm{D}\varphi\,\mathrm{D}\chi\,\exp\left\{\sum_n \int \mathrm{d}x\,\mathrm{d}\tau\,\varphi\hat{O}_0\varphi - \frac{1}{g}\sum_n \int \mathrm{d}x\,\mathrm{d}\tau\,\chi^2 + \sum_n \int \mathrm{d}x\,\mathrm{d}\tau\bar{\psi}[\gamma_0(\partial_\tau - \mu - 2e\varphi) - \mathrm{i}\gamma_1 V_F(\partial_x - \mathrm{i}eA) + (\Delta - \chi)]\psi\right\} \quad (13)$$

where μ is given by $\mu_n = \mu + \pi^2 n^2/2m^* d^2$, m^* is the effective mass of the electron.

Let us obtain the expression for the canonical partition function $Z(N)$. The grand partition function $Z(\mu)$ is expressed through the series expansion

$$Z(\mu) = \sum_N \exp\left(\frac{\mu N}{T}\right) Z(N) \quad (14)$$

which gives

$$Z(N_0)\exp\left(\frac{\mu N_0}{T}\right) = \sum_N \delta_{N,N_0}\exp\left(\frac{\mu N}{T}\right) Z(N) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{d}\lambda\,\exp(\mathrm{i}\lambda N_0)\sum_N Z(N)\exp\left(\frac{\mu N}{T} - \mathrm{i}\lambda N\right) \quad (15)$$

i.e.

$$Z(N_0) = \frac{1}{2\pi}\exp\left(-\frac{\mu N_0}{T}\right)\int_{-\pi}^{\pi} \mathrm{d}\lambda\,Z(\mu - \mathrm{i}\lambda T)\exp(\mathrm{i}\lambda N_0) \quad (16)$$

where $\mu = \mu(N_0)$. Equation (16) allows us to calculate $Z(N_0)$ if one knows $Z(\mu)$.

For spinless electrons, equation (16) solves the problem and, for an electron with spin, one has to add the conservation condition

$$N_0 = N_\uparrow + N_\downarrow \quad (17)$$

to the equation

$$Z(N_0) = Z(N_\uparrow)Z(N_\downarrow). \quad (18)$$

2.2. Partition function in a low-channel limit

After integration over the Grassman variables the partition functions take the form

$$Z(\mu) = \int \mathrm{D}\varphi\,\mathrm{D}\bar{\Delta}\exp\left(-\sum_n \int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\,\varphi\hat{O}_0\varphi\right) \times \exp\left(-\frac{1}{g}\sum_n \int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau(\Delta + \bar{\Delta})^2\right)\exp(\tfrac{1}{2}Sp\log\hat{K}) \quad (19)$$

where

$$\bar{\Delta} = \chi - \Delta \quad (20)$$

$$K = -(\partial_\tau - \mu - 2e\varphi)^2 - V_F^2(\partial_x - \mathrm{i}eA)^2 + 2\mathrm{i}eV_F\gamma_5\partial_x\varphi + 2e\partial_\tau\varphi - \mathrm{i}\gamma_1 V_F\partial_x\bar{\Delta} + \gamma_0\partial_\tau\bar{\Delta} + \bar{\Delta}^2 \quad (21)$$

$\gamma_5 = \gamma_0\gamma_1$, i.e. $(\gamma_0, \mathrm{i}\gamma_1, \gamma_5)$ form a complete set of Pauli matrices, and

$$Sp\hat{Q} = \mathrm{Tr}\left(\sum_n \int \mathrm{d}x\,\mathrm{d}\tau\,\langle x\tau n|\hat{Q}|x\tau n\rangle\right) \quad (22)$$

where symbol Tr is the trace over the matrix indices. In what follows we make use of the representation (see, e.g., [19])

$$Sp\log\hat{K} = -\zeta'(0) - \zeta(0)\log C_R^2 \tag{23}$$

where the generalized zeta function is the sum over the powers of the eigenvalues of the operator $\hat{K}$:

$$\zeta(s) = \sum_{\alpha} \lambda_{\alpha}^{-s} \tag{24}$$

where

$$\hat{K}\eta_{\alpha} = \lambda_{\alpha}\eta_{\alpha} \tag{25}$$

and $\{\eta_{\alpha}\}$ is a complete set of eigenfunctions and C_R is a cut-off.

One can calculate $\zeta(s)$ in the limit of slowly varying fields φ and $\bar{\Delta}$. To obtain the criterion, we present $Sp\log\hat{K}$ in a perturbation series over gradients:

$$Sp\log\hat{K} \cong Sp\log\hat{K}_0 + V_F^2\int \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}\tau\,\mathrm{d}\tau' \sum_{n,n'}\left\{(\partial_x\bar{\Delta})(\partial_{x'}\bar{\Delta}) + \frac{1}{V_F^2}(\partial_\tau\bar{\Delta})(\partial_{\tau'}\bar{\Delta}) + 4e^2(\partial_x\varphi)(\partial_x\varphi') + \frac{4e^2}{V_F^2}(\partial_\tau\varphi)(\partial_{\tau'}\varphi)\right\} G_n(x\tau|x'\tau')G_{n'}(x\tau|x'\tau') \tag{26}$$

where the operator $\hat{K}_0$ is

$$\hat{K}_0 = -(\partial_\tau - \mu - 2e\varphi)^2 - V_F^2(\partial_x - \mathrm{i}eA)^2 + \bar{\Delta}^2. \tag{27}$$

In equation (27), the fields φ and $\bar{\Delta}$ are supposed to be constant; $G_n(x\tau|x'\tau')$ is the Green function of a two-dimensional Helmholtz operator.

In a low-channel limit, G_n scales as

$$x_0 \approx \min\left(L, \frac{V_F}{\bar{\Delta}}\right) \qquad \tau_0 \approx \min\left(\beta, \frac{1}{\bar{\Delta}}\right). \tag{28}$$

The expansion is justified when $e^2/V_F \gg 1$; in this limit the second term on the right-hand side of equation (26) takes the local form

$$\tfrac{1}{2}x_\varphi^2(\partial_x\varphi)^2 + \tfrac{1}{2}\tau_\varphi^2(\partial_\tau\varphi)^2 + \tfrac{1}{2}x_{\bar{\Delta}}^2(\partial_x\bar{\Delta})^2 + \tfrac{1}{2}\tau_{\bar{\Delta}}^2(\partial_\tau\bar{\Delta})^2 \tag{29}$$

where

$$x_\varphi \approx \left(\frac{e^2}{V_F}\right)^{1/2} x_0 \qquad \tau_\varphi \approx \left(\frac{e^2}{V_F}\right)^{1/2}\tau_0 \tag{30}$$

$$x_{\bar{\Delta}} \approx \left(\frac{e^2}{V_F}\right)^{1/2}\begin{cases} x_0 & \text{in a pure ring} \\ x_0\sqrt{\dfrac{\bar{\Delta}}{\Delta}} & \text{in a disordered ring} \end{cases} \tag{31}$$

$$\tau_{\bar{\Delta}} \approx \left(\frac{e^2}{V_F}\right)^{1/2}\begin{cases} \tau_0 & \text{in a pure ring} \\ \tau_0\sqrt{\dfrac{\bar{\Delta}}{\Delta}} & \text{in a disordered ring.} \end{cases} \tag{32}$$

The spatial derivative $\partial_x\mu_{imp}$ can be neglected under the condition $C_{imp}x_0 \ll 1$, where C_{imp} is the concentration of impurities.

Eventually, at $e^2/V_F \gg 1$, the partition function of a low-channel ring takes the form

$$Z(\mu) = \int \mathrm{D}\varphi\, \mathrm{D}\bar{\Delta} \exp\left(- M \int_0^L \mathrm{d}x \int_0^\beta \mathrm{d}\tau\, (\varphi \hat{O}_\varphi \varphi)\right) \times \exp\left(-\frac{M}{g}\int_0^L \mathrm{d}x \int_0^\beta \mathrm{d}\tau (\Delta + \bar{\Delta})^2\right) \exp\left(- M \int_0^L \mathrm{d}x \int_0^\beta \mathrm{d}\tau (\bar{\Delta}\hat{O}_{\bar{\Delta}}\bar{\Delta})\right) \times \exp\left(\frac{1}{2} Sp \log \hat{K}\right) \tag{33}$$

where the $\hat{O}$ operators take the form

$$\hat{O}_{\bar{\Delta}} = \tau_{\bar{\Delta}}\partial_\tau^2 + x_{\bar{\Delta}}\partial_x^2$$
$$\hat{O}_\varphi = \tau_\varphi^2 \partial_\tau^2 + (x_\varphi^2 + x^2)\partial_x^2 + 2x\partial_x. \tag{34}$$

The eigenfunctions of $\hat{O}_\varphi$ are the Legendre functions of imaginary arguments $P_\nu(\mathrm{i}x/x_\varphi)$.

Integration over auxiliary fields is a one-dimensional problem; the non-trivial effect of a finite number of channels is hidden in the $\zeta(s)$.

The generalized zeta function for the operator $\hat{K}$ is

$$\zeta(s) = \sum_{n=0}^{M}\sum_{m=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\left\{\left(2\pi T\left(m+\frac{1}{2}\right) + \mathrm{i}\mu_n\right)^2 + \frac{4\pi^2 V_F^2}{L^2}\left(l - \frac{\Phi}{\Phi_0}\right)^2 + \bar{\Delta}^2\right\}. \tag{35}$$

Here $2\pi T(m + \frac{1}{2})$ is the Matsubara fermion frequency and l is the azimuthal quantum number. It is convenient to perform the summation over channels (over n) after calculation of the one-dimensional $\zeta(s)$.

To calculate the one-dimensional $\zeta(s)$, apply the Poisson summation formula

$$\zeta(s) = (2\pi T)^{-2s}\frac{T}{T_0}\int_{-\infty}^{\infty}\mathrm{d}m\,\mathrm{d}l\,(m^2+l^2+z^2)^{-s} + 2(2\pi T)^{-2s}\frac{T}{T_0}\int_{-\infty}^{\infty}\mathrm{d}m\,\mathrm{d}l \times \sum_{p=1}^{\infty}\frac{\cos(2\pi plT/T_0 + 2\pi p\Phi/\Phi_0)}{(m^2+l^2+z^2)^s} + 2(2\pi T)^{-2s}\frac{T}{T_0}\int_{-\infty}^{\infty}\mathrm{d}m\,\mathrm{d}l \times \sum_{k=1}^{\infty}\frac{\cos[2\pi km - \mathrm{i}(\mu/T)k - k\pi]}{(m^2+l^2+z^2)^s} + 4(2\pi T)^{-2s}\frac{T}{T_0}\int_{-\infty}^{\infty}\mathrm{d}m\,\mathrm{d}l \times \sum_{k=1}^{\infty}\frac{\cos[2\pi km - \mathrm{i}(\mu/T)k - k\pi]\cos(2\pi plT/T_0 + 2\pi p\Phi/\Phi_0)}{(m^2+l^2+z^2)^s} \tag{36}$$

where $Z = \bar{\Delta}/2\pi T$ and $T_0 = V_F/L$.

The function $\zeta(s)$ is calculated in appendix 1.

3. Persistent current at a fixed chemical potential

Making use of equations (A1.2), (A1.3), (A1.5), (A1.10)–(A1.13), (A1.17) and (A1.18), we present the persistent current in the form

$$J(\Phi) = \langle J(\Phi, \bar{\Delta}, \varphi)\rangle \tag{37}$$

where

$$\langle J\rangle = \left[\int \mathrm{D}\varphi\,\mathrm{D}\bar{\Delta}\exp\left(-\int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\ F_0\right) J\,P(\varphi,\bar{\Delta})\right] \times\left[\int \mathrm{D}\varphi\,\mathrm{D}\bar{\Delta}\exp\left(-\int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\ F_0\right) P(\varphi,\bar{\Delta})\right]^{-1} \tag{38}$$

$$F_0 = \frac{\bar{\Delta}^2}{2\pi V_F} M\log\left(\frac{C_R}{\max(\bar{\Delta},2\pi T)}\right) - \frac{1}{2\pi V_F}\sum_{n=0}^{M-1}(\mu_n + 2e\varphi) \tag{39}$$

$$P(\varphi,\bar{\Delta}) = \exp\left(-M\int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\ \left((\bar{\Delta}\hat{O}_{\bar{\Delta}}\bar{\Delta} + \varphi\hat{O}_\varphi\varphi) + \frac{1}{g}(\Delta+\bar{\Delta})^2\right)\right) \tag{40}$$

and

$$J = \begin{cases} 4\pi\dfrac{T}{\Phi_0}\displaystyle\sum_{p=1}^{\infty}\sum_{n=0}^{M-1}\frac{\sin(2\pi p\Phi/\Phi_0)\cos\{[(\mu_n+2e\varphi)/T_0]p\}}{\sinh(\pi pT/T_0)} & \bar{\Delta}\ll T \qquad (41a)\\ \dfrac{4T_0}{\Phi_0}\displaystyle\sum_{p=1}^{\infty}\sum_{n=0}^{M-1}\frac{\sin(2\pi p\Phi/\Phi_0)\cos\{[(\mu_n+2e\varphi)/T_0]p\}}{p} \\ \quad -\dfrac{4T_0}{\Phi_0}\sqrt{\dfrac{\pi T}{2\bar{\Delta}}}\left(\dfrac{T}{T_0}\right)^2\sin\left(2\pi\dfrac{\Phi}{\Phi_0}\right)\exp\left(-\dfrac{\bar{\Delta}}{T}\right) \\ \quad \times\displaystyle\sum_{n=0}^{M-1}\cos\left(\frac{\mu_n+2e\varphi}{T_0}\right) & T\ll\bar{\Delta}\ll T_0 \qquad (41b)\\ 2\pi\dfrac{T_0}{\Phi_0}\sqrt{\dfrac{\pi}{2}}\sin\left(2\pi\dfrac{\Phi}{\Phi_0}\right)\left[\dfrac{1}{2}\sqrt{\dfrac{T_0}{\bar{\Delta}}}\exp\left(-\dfrac{\bar{\Delta}}{T_0}\right)\right. \\ \quad \left.-\dfrac{T}{T_0}\sqrt{\dfrac{T}{\bar{\Delta}}}\left(\dfrac{T^2}{T_0^2}+1\right)^{-3/4}\exp\left(-\bar{\Delta}\sqrt{\dfrac{1}{T^2}+\dfrac{1}{T_0^2}}\right)\right] \\ \quad \times\displaystyle\sum_{n=0}^{M-1}\cos\left(\frac{\mu_n+2e\varphi}{T_0}\right) & T,T_0\ll\bar{\Delta}. \qquad (41c)\end{cases}$$

The summation over spin has already been performed in equations (39) and (41).

The current $J_0(\Phi) = J(\Phi,\bar{\Delta}=\Delta,\varphi=0)$ is simply the persistent current in a gas of non-interacting electrons whose spectrum is

$$\varepsilon_n = (4\pi^2T_0^2n^2+\Delta^2)^{1/2} \tag{42}$$

where n is an integer. The persistent current is sensitive to disorder when $\bar{\Delta}\gg T_0$ (equation (41c)) [22]. When $\bar{\Delta}\ll T_0$, or at high temperatures when $\bar{\Delta}\ll T$, the persistent current reaches its value in a pure metal (equations (41a) and (41b)). In the leading order in $1/L$, the path integral over $\bar{\Delta}$ is defined by its saddle trajectory; the extremum condition at $T\ll\bar{\Delta}$ takes the form

$$\frac{\delta}{\delta\bar{\Delta}}\left\{-\frac{(\Delta+\bar{\Delta})^2}{g} - \frac{\bar{\Delta}^2}{2\pi V_F}\left(\log C_R+\frac{1}{2}\right) + \frac{\bar{\Delta}^2}{2\pi V_F}\log\bar{\Delta}\right\} = 0. \tag{43}$$

The non-trivial solution $\bar{\Delta}=\bar{\Delta}_0$ to (43) exists when

$$\bar{\Delta}_0 \ll \frac{\Delta}{\aleph} \ll \Delta \ll C_R \approx \mu$$

where $\aleph = g/2\pi V_F$, and $\bar{\Delta}_0$ has the form

$$\bar{\Delta}_0 \approx \mu \exp\left(-\frac{\Delta}{\aleph \bar{\Delta}_0}\right). \tag{44}$$

For $\bar{\Delta} \ll T$, one can easily derive the term $\bar{\Delta}^4$ in the free-energy functional (39) and confirm that there exists the critical temperature T_C:

$$T_C = \frac{\bar{\Delta}_0}{\pi} \exp C \tag{45}$$

where C is the Euler constant. For $T > T_C$, $\bar{\Delta}_0 = 0$ and

$$\bar{\Delta}_0(T) \approx \Delta_0 \sqrt{1 - \frac{T}{T_C}} \tag{46}$$

when $T_C - T \ll T_C$.

In a pure ring when $\Delta = 0$, integration over $\bar{\Delta}$ does not affect the persistent current at zero temperature in accordance with the general theorem [15].

In the above calculations we have put the impurity field $\Delta =$ constant. This assumption is justified at a large concentration of Coulomb impurities located in the bulk of a semiconductor (appendix 2). Under this condition which holds in an actual experiment, the persistent current has exponential asymptotes at low temperatures in contrast with a power law specific to a local impurity model [26, 27].

Eventually, the persistent current takes the form

$$J(\Phi) = \int \mathrm{D}\varphi J(\bar{\Delta}_0(T), \varphi, \Phi) \exp\left(- M \int \mathrm{d}x\, \mathrm{d}\tau \left(\varphi \hat{O}_\varphi \varphi + \frac{1}{2\pi V_F} \sum_{n=0}^{M-1} (\mu_n + 2e\varphi)^2 \right) \right). \tag{47}$$

In our model, the operator $\hat{O}_\varphi$ is scale invariant (see equation (34)), and the shift $\mu_n + 2e\varphi \to 2e\varphi$ gives rise to the φ-field mass m_0:

$$m_0^2 = \frac{2e^2}{\pi V_F}. \tag{48}$$

The operator $\hat{O}_\varphi$ transforms into

$$\hat{O} = \partial_t^2 + (1 + y^2)\partial_y^2 + 2y\partial_y - m_0^2 \tag{49}$$

where $y = x/x_\varphi$, $t = \tau/\tau_\varphi$.

The eigenvalues of $\hat{O}$ are determined from the condition of the absence of divergence at $y \to \infty$ [21], i.e.

$$\eta_{n,p} = n(n+1) + \Omega_p^2 - m_0^2 \tag{50}$$

where $\Omega_p = 2\pi p T \tau_\varphi$, and p and n are integers. Integrations over φ are performed by making use of the well known formula [18]

$$\int \mathrm{D}\varphi \exp\left(- \int \mathrm{d}x\, \mathrm{d}\tau (\varphi \hat{O} \varphi + \aleph \varphi) \right) = (\det \hat{O})^{1/2} \exp\left(\tfrac{1}{4} \int \mathrm{d}x\, \mathrm{d}x'\, \mathrm{d}\tau\, \mathrm{d}\tau' \aleph(x, \tau) G(x\tau | x'\tau') \aleph(x', \tau') \right) \tag{51}$$

where

$$G(yt|y't') = \frac{1}{M \tau_\varphi x_\varphi} \sum_{n,p} \frac{\varphi_n(y)\varphi_n(y')}{\eta_{n,p}} \exp[\mathrm{i}\Omega_p(t - t')]. \tag{52}$$

The coefficient $\aleph$ in our case (equation (47)) is

$$\aleph = \pm \mathrm{i} \frac{2e}{V_F} T \tag{53}$$

because

$$\cos\left(\frac{2e\varphi}{T_0}\right) = \cos\left(\aleph \int_0^L \mathrm{d}x \int_0^\beta \mathrm{d}\tau\, \varphi\right). \tag{54}$$

Finally

$$\left\langle \cos\left(\frac{2e\varphi}{T_0}\right)\right\rangle = \exp\left(-\frac{e^2T^2}{V_F^2}\int \mathrm{d}x\, \mathrm{d}x'\, \mathrm{d}\tau\, \mathrm{d}\tau' G(x\tau|x'\tau')\right). \tag{55}$$

When $x_\varphi/L \gg 1$ (a limiting case of a strong Coulomb interaction), only the zero mode $\eta_{0,0} = -m_0^2$ contributes to the Green function (52). The zero-mode eigenfunctions are

$$\varphi_0(x) = \frac{1}{\sqrt{L}}$$

and equation (55) takes the form

$$\left\langle \cos\left(\frac{2e\varphi}{T_0}\right)\right\rangle \cong \exp\left(-\frac{\pi T}{2MT_0}\right) \tag{56}$$

i.e. the φ Coulomb interaction enhances the persistent current.

The crossover temperature at $T \gg \bar{\Delta}_0$ (equation (41*a*)) is

$$T^* = \frac{T_0}{1 - 1/2M}. \tag{57}$$

When $M = 1$ (strictly one-dimensional ring), we restore the doubling of a crossover temperature at strong Coulomb repulsion, first obtained in [13].

Equation (56) shows also that integration over the φ field at zero temperature cancels out the oscillatory dependence on μ and V_g ($\langle\cos(\mu/T_0)\rangle = 1$ in equations (41*b*) and (41*c*)). The bottom line is trivial; the renormalized oscillations frequency is roughly $1/T_0(e^2/V_F + 1)$ which is zero to leading order in $e^2/V_F \gg 1$.

To emphasize the main points, compare the persistent currents in a Coulomb gas and in a free gas (figure 1).

(1) Increasing the number M of channels decreases the non-averaged persistent current of free electrons (equation (41)) by a small factor $M/\sqrt{k_F L} \ll 1$ [23]:

$$J_0(\Phi) \approx J_0(\Phi, M = 1, T)\frac{M}{\sqrt{k_F L}}. \tag{58}$$

The averaged persistent current is proportional to $M^{1/2}$ [4]. The Coulomb interaction prohibits fluctuations in the number of particles in different channels, thus increasing the transverse rigidity of the electron gas. For $M \ll e^2/V_F$ we get

$$J_0(\Phi) \approx M J_0(\Phi, M = 1, T) \exp\left(\frac{\pi T}{2MT_0}\right) \tag{59}$$

i.e. increasing the number of channels enhances the persistent current of interacting electrons proportional to M. The crossover temperature also increases (equation (57)).

(2) Impurities always suppress the persistent current; they define the characteristic temperature T_{imp} below which the persistent current is exponentially suppressed provided that $T_{imp} \gg T_0$. If $T_{imp} \ll T_0$, the disorder suppression of the persistent current is negligible.

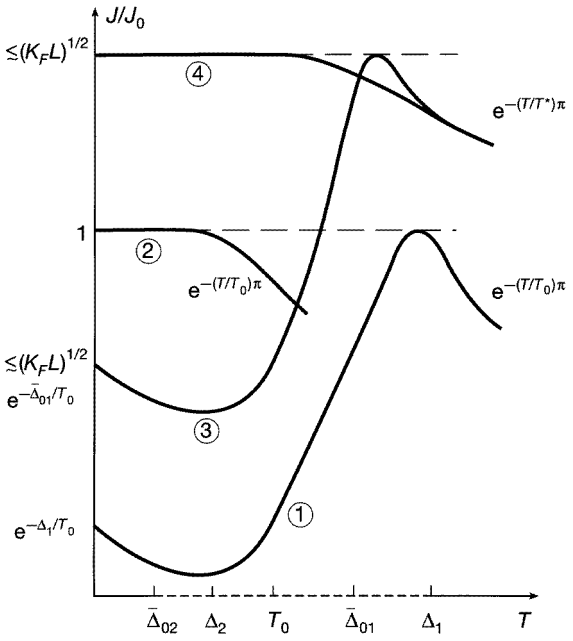

Figure 1. Temperature dependence of the persistent current at various parameters Δ and $\bar{\Delta}_0$ for a fixed chemical potential: curve 1, non-interacting gas, $T_0 \ll \Delta = \Delta_1$; curve 2, non-interacting gas, $\Delta = \Delta_2 \ll T_0$; curve 3, Coulomb gas, $\bar{\Delta}_0 = \bar{\Delta}_{01}$, $T_0 \ll \bar{\Delta}_{01} \ll \Delta$; curve 4, Coulomb gas, $\bar{\Delta}_0 = \bar{\Delta}_{02}$, $\bar{\Delta}_{02} \ll T_0 \ll \Delta$.

In a free gas, $T_{imp} = \Delta$ and, in a Coulomb gas, $T_{imp} = \bar{\Delta}_0 \ll \Delta$ (equation (44)). So, the Coulomb renormalization of T_{imp} favours enhancement of the persistent current.

(3) The persistent current in a free gas manifests an oscillatory dependence over the gate voltage [3, 4, 13]; the pth harmonic of J_0 is proportional to

$$\cos\left(p\pi\frac{N}{M}\right)\cos\left(p\frac{eV_g}{T_0}\right) \tag{60}$$

where N is the number of electrons in the ring at a given electrochemical potential. A strong Coulomb interaction cancels out this dependence.

4. Persistent current at a fixed number of particles

At a fixed number of electrons, equation (38) takes the form

$$\langle J(\Phi)\rangle_{N_0} = \left[\int \mathrm{D}\varphi\,\mathrm{D}\bar{\Delta}\int_{-\pi}^{\pi}\mathrm{d}\lambda\,\exp(\mathrm{i}\lambda N_0)\exp\left(-\int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\,F_0\right)JP(\varphi,\bar{\Delta})\right] \times\left[\int \mathrm{D}\varphi\,\mathrm{D}\bar{\Delta}\int_{-\pi}^{\pi}\mathrm{d}\lambda\,\exp(\mathrm{i}\lambda N_0)\exp\left(-\int_0^L \mathrm{d}x\int_0^\beta \mathrm{d}\tau\,F_0\right)P(\varphi,\bar{\Delta})\right]^{-1} \tag{61}$$

where μ in equations (39) and (41) is substituted by $\mu - \mathrm{i}\lambda T$ (equation (16)).

One has to be careful at this point because the naive shift $\varphi \to \varphi + \mu/2e - \mathrm{i}\lambda T/2e$ causes $Z(N_0)$ to become zero, and so the order of operations is relevant; first, one has to carry out the integration over λ and then to shift the $\varphi-$ field in the functional integral.

We make use of the asymptotic equality

$$\left[\int_{-\pi}^{\pi} d\lambda \exp\left(i\lambda N_0 + \frac{1}{2\pi T T_0}\sum_n (\mu_n + 2e\varphi - i\lambda T)^2\right)\cos\left(\frac{\mu_n + 2e\varphi - i\lambda T}{T_0}\right)\right] \times\left[\int_{-\pi}^{\pi} d\lambda \exp\left(i\lambda N_0 + \frac{1}{2\pi T T_0}\sum_n (\mu_n + 2e\varphi - i\lambda T)^2\right)\right]^{-1} \cong \begin{cases} \cos\left(\pi\dfrac{N_0}{M}\right) & T \ll T_0 \\ \exp\left(\pi\dfrac{T}{2T_0}\right) & T_0 \ll T. \end{cases} \tag{62}$$

Equation (62) shows that at low temperatures the non-trivial parity effect takes place; the sign of a persistent current is defined by the occupation of a single channel. For an electron with spin, the parity effect is ill defined because the persistent current is proportional to

$$\langle j(\Phi)\rangle_{N_0} \approx \cos\left(\pi\frac{N_\uparrow}{M}\right) + \cos\left(\pi\frac{N_\downarrow}{M}\right)$$
$$N_\downarrow + N_\uparrow = N_0 \tag{63}$$

and the difference $N_\uparrow - N_\downarrow$ is a free parameter, which has to be additionally fixed by the magnetic field.

At high temperatures the parity effect is smeared out and the crossover temperature doubles [12, 25].

The integrand J in equation (61) turns out to be independent of the φ field; therefore $\langle J(\Phi)\rangle_{N_0}$ is proportional to the number of channels.

Integration over the $\bar{\Delta}$ field gives the same result as in the case of the fixed chemical potential.

So, the only principal distinction between the persistent currents in a free gas and in a Coulomb gas is the value of the localization gap: Δ in a free gas, and $\bar{\Delta}_0(T) \ll \Delta$ in a Coulomb gas.

5. Conclusion

In this paper we have studied analytically the persistent currents at finite temperatures in a gas of electrons interacting via a long-range Coulomb field in a disordered thin ring.

The Coulomb interaction always enhances the persistent current in a twofold way: firstly its $2k_F$ component suppresses intrinsic disorder and the renormalized localization gap becomes zero at $T = T_C$; secondly its zero-momentum component increases the 'transverse rigidity' of a gas: the persistent current turns out to be proportional to the number of transverse channels; the crossover temperature increases at a fixed chemical potential.

The effect of Coulomb back scattering exists at both a fixed chemical potential and a fixed number of electrons in the ring. The forward scattering does not affect the persistent current at a fixed number of particles.

At a fixed chemical potential the strong Coulomb interaction cancels out oscillations over gate voltage which exist in a free gas. At a fixed number of particles a non-trivial parity effect emerges; the sign of a persistent current is defined by the occupation of a single channel.

At certain values of parameters, the temperature dependence of the persistent current is non-monotonic.

Acknowledgments

I acknowledge I V Krive for numerous fruitful discussions. This work was initiated by discussions held at the Workshop on Strongly Correlated Electrons, ICTP, Trieste, 1995. I acknowledge Yu Lu for the organization of the Workshop. I acknowledge P Monceau for his hospitality during my stay at CRTBT. I am indebted to Daniele Devillers and Yurij Pershin for technical assistance.

This work was partially supported by the Joint Grant U2K200 awarded by the Government of Ukraine and the ISF.

Appendix 1

Here we calculate the one-dimensional $\zeta(s)$ (equation (36)) and functions $\zeta(0)$ and $\zeta'(0)$. Trivial integration yields

$$\begin{aligned}\zeta(s) = (2\pi T)^{-2s}\frac{T}{T_0}\Bigg\{&\pi\frac{z^{2-2s}}{s-1} + 4\pi z^{2-2s}\frac{1}{\Gamma(s)}\sum_{p=1}^{\infty}\cos\left(2\pi p\frac{\Phi}{\Phi_0}\right)\left(\pi\frac{T}{T_0}pz\right)^{s-1}K_{1-s}\\ &\times\left(2\pi\frac{T}{T_0}pz\right) + 4\pi z^{2-2s}\frac{1}{\Gamma(s)}\sum_{k=1}^{\infty}(-1)^k\cosh\left(\frac{\mu}{T}k\right)(\pi kz)^{s-1}K_{1-s}(2\pi kz)\\ &+8\pi z^{2-2s}\frac{1}{\Gamma(s)}\sum_{\substack{p=1\\k=1}}^{\infty}\cos\left(2\pi p\frac{\Phi}{\Phi_0}\right)\frac{(-1)^k\cosh[(\mu/T)k]}{(\pi z\sqrt{k^2+p^2T^2/T_0^2})^{1-s}}K_{1-s}\\ &\times\left(2\pi z\sqrt{k^2+p^2\frac{T^2}{T_0^2}}\right)\Bigg\}\end{aligned} \tag{A1.1}$$

where $\Gamma(s)$ is the gamma function and $K_\nu(y)$ is the McDonald function. It is easy to see that

$$\frac{T_0}{T}\zeta(0) = -\pi z^2 \tag{A1.2}$$

and

$$\begin{aligned}\frac{T_0}{T}\zeta'(0) = -\pi z^2 + \pi z^2\log z^2 + \pi z^2\log[(2\pi T)^2] + 4\frac{T_0}{T}z\sum_{p=1}^{\infty}\frac{\cos(2\pi p\Phi/\Phi_0)}{p}K_1\left(2\pi\frac{T}{T_0}pz\right)\\ +4z\sum_{k=1}^{\infty}\frac{(-1)^k\cosh[(\mu/T)k]}{k}K_1(2\pi kz)\\ +8z\sum_{p=1}^{\infty}\cos\left(2\pi p\frac{\Phi}{\Phi_0}\right)\frac{(-1)^k\cosh[(\mu/T)k]}{\sqrt{k^2+p^2T^2/T_0^2}}K_1\left(2\pi z\sqrt{k^2+p^2\frac{T^2}{T_0^2}}\right).\end{aligned} \tag{A1.3}$$

First, we calculate the asymptotics of (A1.3) at $z \ll 1$, making use of the equation

$$K_1(y)|_{y\ll 1} \approx \frac{1}{y} + \frac{y}{2}\log y \tag{A1.4}$$

and recalling that $\mu/T \gg 1$.

We get [20]

$$\frac{1}{2\pi z}\sum_{k=1}^{\infty}\frac{\cosh[(\mu/T)k]}{k^2}(-1)^k+\pi z\log z\sum_{k=1}^{\infty}\cosh\left(\frac{\mu}{T}k\right)(-1)^k$$
$$=-\frac{\pi}{24z}-\frac{1}{8\pi z}\left(\frac{\mu}{T}\right)^2-\frac{1}{2}\pi z\log z+\mathrm{O}\left[\exp\left(-\frac{\mu}{T}\right)\right]. \tag{A1.5}$$

To calculate the sum

$$F(\mu)=\sum_{k=1}^{\infty}\frac{\cosh[(\mu/T)k]}{k^2+a^2}(-1)^k \tag{A1.6}$$

we compose the differential equation

$$\frac{\mathrm{d}^2F}{\mathrm{d}\mu^2}+a^2F=-\tfrac{1}{2} \tag{A1.7}$$

with the boundary conditions

$$F(0)=\frac{\pi}{2a}\frac{1}{\sinh(\pi a)}-\frac{1}{2a^2}$$
$$\left.\frac{\mathrm{d}F}{\mathrm{d}\mu}\right|_{\mu=0}=0. \tag{A1.8}$$

We get

$$F\left(\frac{\mu}{T}\right)=\left(F(0)+\frac{1}{2a^2}\right)\cos\left(\frac{\mu}{T}a\right)-\frac{1}{2a^2}. \tag{A1.9}$$

Making use of equation (A1.9) we eventually obtain

$$\left(\frac{T_0}{T}\right)^2\sum_{p=1}^{\infty}\frac{\cos(2\pi p\Phi/\Phi_0)}{p^2}+2\sum_{\substack{p=1\\k=1}}^{\infty}\cos\left(2\pi p\frac{\Phi}{\Phi_0}\right)\frac{(-1)^k\cosh[(\mu/T)k]}{k^2+p^2T^2/T_0^2}$$
$$=\frac{T_0}{T}\sum_{p=1}^{\infty}\frac{\cos(2\pi p\Phi/\Phi_0)}{p\sinh(\pi pT/T_0)}\cos\left(\frac{\mu}{T_0}p\right). \tag{A1.10}$$

Equations (A1.5) and (A1.10) solve the problem when $\bar{\Delta}\ll T$, T_0. Now we turn to the asymptotics of equation (A1.3) at $\bar{\Delta}\sqrt{1/T^2+1/T_0^2}\gg 1$. To calculate the asymptote of the fifth term in equation (A1.3), we make use of the integral representation [29]

$$K_1(2\pi kz)=\frac{2\pi z}{k}\int_0^{\infty}\mathrm{d}t\,\frac{\cos(kt)}{(t^2+4\pi^2z^2)^{3/2}}. \tag{A1.11}$$

We get

$$\sum_{k=1}^{\infty}\frac{1}{k^2}(-1)^k\cos(tk)\cos\left(\frac{\mu}{T}k\right)\cong-\frac{1}{2}\frac{\mu}{T}+\frac{3}{16}\cos t\exp\left(-\frac{\mu}{T}\right) \tag{A1.12}$$

and

$$4z\sum_{k=1}^{\infty}\frac{(-1)^k\cosh[(\mu/T)k]}{k}K_1(2\pi zk)\cong-\frac{1}{2\pi}\left(\frac{\mu}{T}\right)^2+\mathrm{O}(\exp(-2\pi z)). \tag{A1.13}$$

Calculation of the last term in equation (A1.3) is slightly more cumbersome. We present it in the form

$$-\frac{4z}{\pi}\sum_{p=1}^{\infty}\cos\left(2\pi p\frac{\Phi}{\Phi_0}\right)\frac{\mathrm{d}}{\mathrm{d}z}\sum_{k=1}^{\infty}\frac{(-1)^k\cosh[(\mu/T)k]}{k^2+p^2T^2/T_0^2}K_0\left(2\pi z\sqrt{k^2+p^2\frac{T^2}{T_0^2}}\right) \tag{A1.14}$$

and make use of the integral representation [29]

$$K_0\left(2\pi z\sqrt{k^2+p^2\frac{T^2}{T_0^2}}\right)=\sqrt{\frac{\pi}{2}}\left(2\pi zp\frac{T}{T_0}\right)^{1/2}\int_1^\infty \mathrm{d}x\,(x^2-1)^{-1/4}\exp(-2\pi zkx)J_{-1/2}$$
$$\times\left(2\pi zp\frac{T}{T_0}\sqrt{x^2-1}\right) \tag{A1.15}$$

where $J_\nu(y)$ is Bessel function.

The summation over k in (A1.14) is performed explicitly, and we obtain at $z\gg 1$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\cosh[(\mu/T)k]}{k^2+a^2}\exp(-2\pi zkx)\cong-\frac{1}{2a^2}+\left(\frac{1}{2a^2}-\frac{\exp(-2\pi zx)}{a^2+1}\right)\cos\left(\frac{\mu}{T}a\right). \tag{A1.16}$$

By substituting (A1.16) into (A1.15) and performing integration over x, we obtain the following expression for the sum of the third and last terms in (A1.3):

$$\left(\frac{T_0}{T}\right)^2\sum_{p=1}^{\infty}\frac{\cos(2\pi p\Phi/\Phi_0)\cos[(\mu/T_0)p]}{p^2}-\sqrt{\frac{\pi}{2}}\sqrt{\frac{T}{\bar\Delta}}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\cos\left(\frac{\mu}{T_0}\right)\exp\left(-\frac{\bar\Delta}{T}\right) \tag{A1.17}$$

when $T\ll\bar\Delta\ll T_0$, and

$$\sqrt{\frac{\pi}{2}}\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\cos\left(\frac{\mu}{T_0}\right)\left[\frac{T_0}{2T}\sqrt{\frac{T_0}{\bar\Delta}}\exp\left(-\frac{\bar\Delta}{T_0}\right)\right.$$
$$\left.-\sqrt{\frac{T}{\bar\Delta}}\frac{1}{(T^2/T_0^2+1)^{3/4}}\exp\left(-\bar\Delta\sqrt{\frac{1}{T^2}+\frac{1}{T_0^2}}\right)\right] \tag{A1.18}$$

when $T,\ T_0\ll\bar\Delta$.

Appendix 2

When rewriting the Hamiltonian (2) in terms of one-dimensional fields ψ, (equation (3)), the back-scattering component $\Delta(x)$ of the impurity potential $V(x)$ is represented in the form of a wave packet of width $|k|-2k_F\approx\bar\Delta/V_F$:

$$\Delta(x)=\sum_k V_k\cos((k-2k_F)x) \tag{A2.1}$$

where V_k are the Fourier harmonics of $V(x)$. Consider Coulomb impurities distributed in the bulk of the semiconductor at points $\boldsymbol{r}_i$. For a high concentration of impurities when $\langle r\rangle\ll L$, where $\langle r\rangle$ is the average distance between impurities, $V(x)$ takes the form

$$V(x)\cong\frac{e^2}{\varepsilon_0\langle r\rangle}\sum_{(y_i,z_i)}\log\left|\frac{L+x+\sqrt{(L+x)^2+\rho_i^2}}{L-x+\sqrt{(L-x)^2+\rho_i^2}}\right| \tag{A2.2}$$

where $\rho_i=(y_i,z_i)$ and ε_0 is the bulk dielectric permeability.

The logarithm is nearly a constant, and

$$V_{2k_F}\approx\frac{e^2}{\varepsilon_0}\frac{n_{imp}}{(k_F\langle r\rangle)^2} \tag{A2.3}$$

where $n_{imp} \approx (L/\langle r\rangle)^3$ is the number of impurities in the approximate volume L^3, $k_F\langle r\rangle \gg 1$. When $L\bar{\Delta}/V_F \leqslant 1$, $\Delta(x)$ is almost independent of x and is of the order of

$$\Delta \approx \frac{e^2}{\varepsilon_0\langle r\rangle}\frac{n_{imp}}{(k_F\langle r\rangle)^2}. \tag{A2.4}$$

Recalling equation (44), the criterion for $\Delta =$ constant when $e^2/V_F \gg 1$ takes the form

$$\frac{L}{\langle r\rangle} \ll (k_F\langle r\rangle)^{1/2}\left[\log\left(\frac{\mu}{\bar{\Delta}}\right)\right]^{1/4}. \tag{A2.5}$$

This inequality holds easily for small rings.

References

[1] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
[2] Krive I V and Rozhavsky A S 1992 *Int. J. Mod. Phys.* B **6** 1255
[3] Kulik I O 1970 *Zh. Eksp. Teor. Fiz.* **11** 407 (Engl. Transl. 1970 *JETP Lett.* **11** 275)
[4] Buttiker M, Imry Y and Landauer R 1983 *Phys. Lett.* **96A** 365
Cheung H-F, Gefen Y, Riedel E K and Shin W-H 1988 *Phys. Rev.* B **37** 6050
Cheung H-F and Riedel E K 1989 *Phys. Rev.* B **40** 9498
von Oppen F and Riedel E K 1991 *Phys. Rev. Lett.* **66** 84
Altshuler B L, Gefen Y and Imry Y 1991 *Phys. Rev. Lett.* **66** 88
Schmid A 1991 *Phys. Rev. Lett.* **66** 80
Cheung H-F, Reidel E K and Gefen Y 1989 *Phys. Rev. Lett.* **62** 587
[5] Levy L P, Dolan G, Dunsmuir J and Bouchiat H 1990 *Phys. Rev. Lett.* **64** 2074
Levy L P 1991 *Physica* B **169** 224
[6] Chaudrasekhar V, Webb R A, Brady M J, Ketchen M B, Gallagher W and Kleinsasser A 1991 *Phys. Rev. Lett.* **67** 3578
[7] Abrahams M and Berkovitz R 1993 *Phys. Rev. Lett.* **79** 1509
[8] Bouzerar G, Poilblanc D and Montambaux G 1994 *Phys. Rev.* B **49** 8258
[9] Chakraborty T and Pietilainen P 1995 *Phys. Rev.* B **52** 1932
[10] Chakraborty T 1992 *Comments Condes. Matter Phys.* **16** 35
[11] Giamarchi G and Sriram Shastry B 1995 *Phys. Rev.* B **51** 10 915
[12] Loss D 1992 *Phys. Rev. Lett.* **69** 343
[13] Krive I V, Sandstrom P, Shechter R I and Johnson M 1996 unpublished
[14] Zvyagin A A and Krive I V 1995 *Fiz. Nizk. Temp.* **21** 688 (Engl. Transl. 1995 *Low Temp. Phys.*)
[15] Muller-Groeling A, Weidenmuller H A and Lewenkopf C H 1993 *Europhys. Lett.* **22** 193
Muller-Groeling A and Weidenmuller H A 1994 *Phys. Rev.* B **49** 4752
[16] Mailly D, Chapelier C and Benoit A 1993 *Phys. Rev. Lett.* **70** 2020
Mailly D, Chapelier C and Benoit A 1994 *Physica* B **197** 514
[17] Krive I V and Pozhavsky A S 1987 *Sov. Phys.– Usp.* **30** 370
[18] Popov V S 1976 *Continual Integrals in Quantum Field Theory and Statistical Physics* (Moscow: Atomizdat)
[19] Krive I V, Pozhavsky A S and Kulik I O 1986 *Fiz. Nizk. Temp.* **12** 1223 (Engl. Transl. 1986 *Sov. J. Low Temp. Phys.* **12** 635)
[20] Prudnikov A P, Brychkov Yu A and Marichev O I 1986 *Integrals and Series* vol 1 *Elementary Functions* (New York: Gordon and Breach)
[21] Kamke E 1956 *Differential Gleichungen* (Leipzig) (1971 (Moscow: Nauka) (in Russian))
[22] Kulik I O, Rozhavsky A S and Bogachek E N 1988 *JETP Lett.* **47** 303
[23] Cheung H-F, Gefen Y and Reidel E K 1988 *IBM J. Res. Dev.* **32** 359
[24] Kamal M, Musslimani Z H and Auerbach A 1995 *J. Physique* **15** 1487
[25] Loss D and Goldbart P 1991 *Phys. Rev.* B **43** 13 762
[26] Kane C L and Fisher M P A 1992 *Phys. Rev.* B **46** 15 233
[27] Glazman L I, Ruzin I M and Shklovskii B I 1992 *Phys. Rev.* B **45** 8454
[28] Krive I V, Sandstrom P, Shekhter R I, Girvin S M and Jonson M 1995 *Phys. Rev.* B **52** 16 451
[29] Gradshtein I S and Ryzhik I M 1972 *Tables of Integrals, Sums, Series and Products* (Moscow: Nauka)